BULLETIN OF THE ASTRONOMICAL INSTITUTES
OF THE NETHERLANDS.

1930 May 26 **Volume V.** **No. 185.**

COMMUNICATION FROM THE OBSERVATORY AT LEIDEN.

On the magnitudes, diameters and distances of the extragalactic nebulae, and their apparent radial velocities, by *W. de Sitter.*

1. Lately several radial velocities of extragalactic nebulae have been published, all of which are large and positive. This makes the question of the distances of these objects of particular importance. For a few spirals, elliptical and irregular nebulae distances have been determined from the variable stars or novae contained in them, or from the brightest stars, or by other means, but for the great majority the distance can only be determined from the apparent total magnitude or the apparent diameter. If we suppose that the spreading in the absolute magnitudes and the linear diameters is small, and that there is no absorption of light in space, these two determinations must agree, and we must have $r_m = r_d$, the distances as determined from the magnitude and from the diameter being respectively:

$$\text{(1)} \qquad \log r_m = \frac{m}{5} - \frac{M}{5}, \qquad \log r_d = \log D - \log d,$$

and consequently

$$\text{(2)} \qquad H = \frac{m}{5} + \log d = \frac{M}{5} + \log D = \text{constant},$$

where m, M are the apparent and absolute total magnitudes and d, D the apparent and linear diameters.

The first step in the present investigation was to test the relation (2), and to derive, for the various types of nebulae, a relation of the form

$$H = \alpha + \beta \log d,$$

or

$$\text{(3)} \qquad \frac{m}{5} = \alpha - (1-\beta) \log d.$$

If (2) is satisfied, we must find $\beta = 0$. The investigation was based on the data contained in HUBBLE's paper of 1926 [1]), in which he gives the apparent visual magnitudes (m_T) and the logarithms of the diameters ($\log d$) of a large number of nebulae.

2. The most serious difficulty in the derivation of the linear formulas (3) is the selection, which undoubtedly exists for the faint and small nebulae, making the average diameter of the faintest nebulae too large, and the average brightness of the smallest ones too great. The same thing occurs, in the opposite direction, and in a smaller degree, at the bright and large end of the curve. One remedy against this would be to reject these extreme objects, but this would considerably reduce the weight of the determination, so much so that in some cases it would become practically impossible. The way in which the material has been treated is the following. First the nebulae of each class were arranged in order of increasing brightness, and the means of the magnitude and the logarithm of the diameter of every ten (or another number) successive nebulae in this arrangement were formed. Then the nebulae were arranged in order of increasing diameter, and an equal number of means was again formed. Then the mean was taken of the corresponding means in the two arrangements, and through these means a straight line was drawn. For the computation of these straight lines the method of HERTZSPRUNG (*Leiden Annals*, Vol. XIV, Part. **1**, page 5) was used. If two series of observed quantities x and y are to be connected by a formula

$$y - y_0 = (x - x_0) \tan \theta,$$

x_0 and y_0 being the weighted means of x and y respectively (the weights used in forming these means being the same for x and y), we have

$$\cot 2\theta = \frac{\Sigma p\,(x-x_0)^2 - \Sigma p\,(y-y_0)^2}{2\,\Sigma p\,(x-x_0)\,(y-y_0)},$$

the weights p being the same as before. The probable error of the angle θ is given by

[1]) *Extra-galactic Nebulae*, by EDWIN HUBBLE, *Ap. J.* LXIV, pp. 321—369, *Contr. Mt. Wilson*, Vol. XIV, No. 324.

$$\mu_\theta^2 = \frac{\Sigma p\,(x-x_0)^2 + \Sigma p\,(y-y_0)^2}{4\,[\Sigma p\,(x-x_0)(y-y_0)]^2 + [\Sigma p\,(x-x_0)^2 - \Sigma p\,(y-y_0)^2]^2}\cdot\mu^2,$$

μ being the probable error of one value of $x-x_0$ or $y-y_0$ [1]), or $\cos\theta \times$ the p. e. of unit weight of one residual $(y-y_0)-(x-x_0)\tan\theta$.

3. *Spiral nebulae.* For the application of this method the following means were formed, the first, or left hand, half of the tables I*A* to VI giving the arrangement in order of magnitude, and the second half the arrangement in order of diameter. The general means are given in the tables I*B* to III*B*.

TABLE I*A*. Spirals *Sa*.

Nr.	$m/5$	log d	Nr.	log d	$m/5$
5	2·580	+·198	5	—·054	2·452
12	2·452	·210	11	+·132	2·428
15	2·362	·334	10	·297	2·334
9	2·246	·458	15	·467	2·322
8	2·080	·511	8	·671	2·184

The first mean in both arrangements was omitted, the others were combined with equal weights, giving:

TABLE I*B*. Spirals *Sa*.

$m/5$	log d
2·440	+·171
2·348	·316
2·284	·462
2·132	·591

From these we find

$$(4)\quad (Sa)\quad \frac{m}{5} = 2{\cdot}301 - (0{\cdot}66 \pm {\cdot}06)\,(\log d - {\cdot}385).$$

TABLE II*A*. Spirals *Sb*.

Nr.	$m/5$	log d	Nr.	log d	$m/5$
			4	—·142	2·600
15	2·599	+·067	9	—·003	2·538
11	2·489	·345	11	+·193	2·438
19	2·388	·383	12	·359	2·377
9	2·236	·703	11	·506	2·347
6	2·097	·660	8	·676	2·200
6	1·870	·797	9	·879	2·027
3	1·693	1·067	5	1·172	2·068
1	1·00	2·25	1	2·25	1·00

The lowest line is the Andromeda nebula, N.G.C. 224.

[1]) The preliminary reduction of x and y to the same weight can be omitted, as it has no influence on the result.

These were combined in two different ways. First the first, seventh and eighth means on the left, and the first, second, eighth and ninth on the right were omitted, the fifth and sixth on the left and the sixth and seventh on the right being combined to one mean. This gave:

TABLE II*B*. Spirals *Sb*.

$m/5$	log d
2·463	+·269
2·382	·371
2·292	·604
2·089	·758

From these we found

$$(5a)\quad (Sb)\quad \frac{m}{5} = 2{\cdot}309 - (0{\cdot}70 \pm {\cdot}11)\,(\log d - {\cdot}500)$$

Then the first mean of Table II*A* on the left and the first and second on the right were combined to one mean, and also the fifth, sixth and seventh on the left with weights 2, 2, 1 and the sixth, seventh and eighth on the right with the same weights. The Andromeda nebula was kept by itself. This gave

TABLE II*C*. Spirals *Sb*.

$m/5$	log d
2·579	+·014
2·463	·269
2·382	·371
2·292	·604
2·047	·830
1·00	2·25

From those we find, assigning weight $\frac{1}{4}$ to the last line,

$$(5\,b)\quad (Sb)\quad \frac{m}{5} = 2{\cdot}287 - (0{\cdot}745 \pm {\cdot}03)\,(\log d - {\cdot}505).$$

The coefficient of (5 *b*) is evidently more reliable than of (5 *a*), but for the zero point the values of (5 *a*) may be preferable.

TABLE III*A*. Spirals *Sc*.

Nr.	$m/5$	log d	Nr.	log d	$m/5$
9	2·660	+·403	5	—·042	2·512
9	2·614	·421	11	+·150	2·500
15	2·530	·589	17	·300	2·452
19	2·478	·414	15	·391	2·422
16	2·408	·430	15	·493	2·406
18	2·364	·528	16	·598	2·458
9	2·280	·476	13	·688	2·248
7	2·114	·771	7	·790	2·478
7	2·000	·714	7	·952	2·304
4	1·830	1·165	6	1·080	1,980
2	1·440	1·43	3	1·487	1·746

These were combined into means as indicated by the brackets, using relative weights equal to the numbers of nebulae.

This gives:

TABLE III *B*. Spirals *Sc*.

$m/5$	$\log d$
2·570	+ ·251
2·491	·444
2·450	·402
2·407	·462
2·411	·563
2·268	·664
1·945	1·032

From these we find

$$(6)\quad (Sc)\quad \frac{m}{5} = 2{\cdot}363 - (0{\cdot}82 \pm {\cdot}05)\,(\log d - {\cdot}545).$$

TABLE IV. Spirals *SBa*.

Nr.	$m/5$	$\log d$	Nr.	$\log d$	$m/5$
4	2·585	— ·025	4	— ·048	2·546
8	2·423	+ ·175	8	+ ·161	2·581
9	2·249	·330	8	·326	2·270
5	2·144	·528	6	·533	2·164

The corresponding means were combined as they stand, so that it is not necessary to print the general means, and Table IV *B* is omitted. From the combined means we find:

$$(7)\quad (SBa)\quad \frac{m}{5} = 2{\cdot}350 - (0{\cdot}75 \pm {\cdot}03)\,(\log d - {\cdot}247).$$

The numbers of spiral nebulae of the classes *SBb* and *SBc* given in HUBBLE's tables are too small for a reliable determination of the correlation between $m/5$ and $\log d$. Making three means in both arrangements we find, however

TABLE V*a*. Spirals *SBb*.

Nr.	$m/5$	$\log d$	Nr.	$\log d$	$m/5$
5	2·48	+ ·11	5	+ ·05	2·40
6	2·32	·38	5	·27	2·38
5	2·10	·44	6	·54	2·20

Combining the means as they are, we find

$$(8a)\quad (SBb)\quad \frac{m}{5} = 2{\cdot}31 - (0{\cdot}71 \pm {\cdot}10)\,(\log d - {\cdot}30).$$

If we make only two means in both arrangements we have

TABLE V *b*. Spirals *SBb*.

Nr.	$m/5$	$\log d$	Nr.	$\log d$	$m/5$
8	2·54	+ ·19	8	+ ·12	2·39
8	2·17	·45	8	·51	2·21

From these we find

$$(8b)\quad (SBb)\quad \frac{m}{5} = 2{\cdot}32 - 0{\cdot}88\,(\log d - {\cdot}32).$$

For the class *SBc* we have similarly

TABLE VI *a*. Spirals *SBc*.

Nr.	$m/5$	$\log d$	Nr.	$\log d$	$m/5$
5	2·60	+ ·40	5	+ ·31	2·52
5	2·40	·50	5	·49	2·31
5	2·14	·63	5	·73	2·30

$$(9a)\quad (SBc)\quad \frac{m}{5} = 2{\cdot}38 - (1{\cdot}08 \pm {\cdot}15)\,(\log d - {\cdot}51).$$

TABLE VI*b*. Spirals *SBc*.

Nr.	$m/5$	$\log d$	Nr.	$\log d$	$m/5$
8	2·51	+ ·42	7	+ ·35	2·46
7	2·19	·62	8	·65	2·29

$$(9b)\quad (Sbc)\quad \frac{m}{5} = 2{\cdot}36 - 0{\cdot}92\,(\log d - {\cdot}51).$$

The formulas (8 *b*) and (9 *b*) have been adopted.

4. Collecting the result for the spiral nebulae, we have the following values of the coefficients of the linear equations

$$(3)\qquad \frac{m}{5} = \left(\frac{m}{5}\right)_0 - (1-\beta)\,(\log d - \log d_0)$$

TABLE VII.

Spiral nebulae. Correlation between m and $\log d$.

Class	Number	$(1-\beta)$	$\left(\frac{m}{5}\right)_0$	$(\log d)_0$	$\delta\left(\frac{m}{5}\right) +$ $\cdot 75\,\delta(\log d)$	Adopted $d(m/5)$	Adopted $\delta(\log d)$
Sa	44	0·66 ± ·06	2·30	+ ·38⁵	+ ·09⁵	+ ·05	+ ·06
Sb	51 (70)	0·74⁵ ± ·03	2·31	·50	·00	·00	·00
Sc	115	0·82 ± ·05	2·36	·54⁵	— ·09	— ·05	— ·05
SBa	26	0·75 ± ·03	2·35	·25	+ ·15	+ ·09	+ ·08
SBb	16	0·88 ± ·10	2·32	·32	+ ·12⁵	+ ·07	+ ·07
SBc	15	0·92 ± ·15	2·36	·51	— ·06	— ·03	— ·04

For *Sb* the coefficient $1-\beta$ has been taken from the formula (5 *b*), based on all nebulae, but $(m/5)_0$ and $(\log d)_0$ from (5 *a*), which are thus the centre of gravity of the nebulae of intermediate magnitudes and sizes.

For *SBb* and *SBc* the formulas (8*b*) and (9*b*) were adopted, assigning to $1-\beta$ the probable errors of (8*a*) and (9*a*). The second column gives the number of objects used in the derivation of the formulas. Both for the ordinary and for the barred spirals there is an apparent progression of the value of $1-\beta$ with the "age" of the nebulae. It is, however very uncertain whether this is real. It appears doubtful whether the probable errors as derived from the residuals can be taken as a true measure of the reliability of the resulting values of $1-\beta$, but even if they were they would not exclude the possibility of the true value being the same for all spirals. On the other hand the deviation of all the coefficients, excepting those for the classes *SBb* and *SBc*, which depend on a small number of objects, from the theoretical value 1 seems to be certainly real. There are no spirals of the classes *SBb* and *SBc* of which either distances or radial velocities have been determined. For all the other classes the result of the further discussion will not be

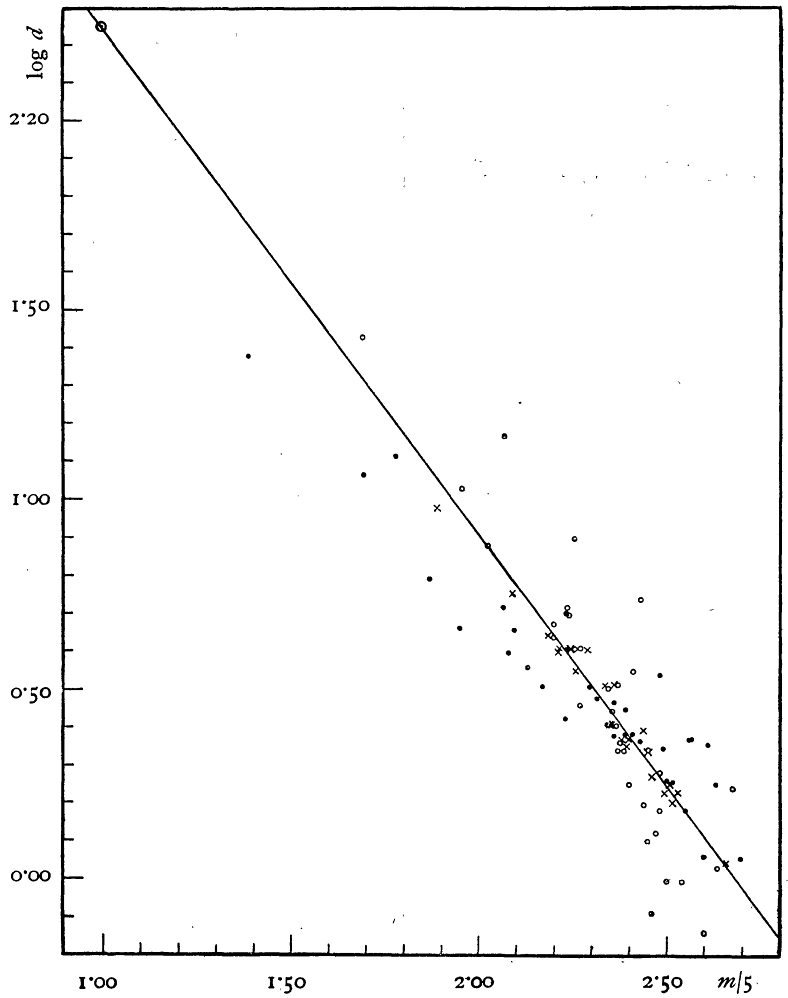

FIGURE 1. Spiral Nebulae. Conclation between $m/5$ and log d.

materially altered if we use the same value of $1-\beta$ for all of them. I have therefore adopted the value

$$1-\beta = 0{\cdot}75, \qquad \beta = 0{\cdot}25$$

for all spirals. Then we can reduce them all to the same formula by applying constant corrections $\delta\,(m/5)$ and $\delta\,(\log d)$ to the observed magnitudes and diameters. These corrections correspond to a parallel displacement of the curve, and can thus be arbitrarily distributed over $m/5$ or $\log d$. HUBBLE (*l.c.* TABLE VIII, p. 25) throws them all on to $\log d$. It has appeared preferable to distribute them equally. The sixth column gives the value of $\delta\,(m/5) + 0{\cdot}75\,\delta\,(\log d)$, where $\delta\,(m/5)$ and $\delta\,(\log d)$ are the corrections required to make the values of $(m/5)_0$ and $(\log d)_0$ for the several classes equal to those for *Sb*. The adopted corrections given in the last two columns satisfy the condition that $\delta\,(m/5) + \delta\,(\log d)$ is equal to the value as given in

the sixth column. The formula for all spirals thus becomes

$$(10)\quad \frac{m}{5} + \delta\left(\frac{m}{5}\right) = 2{\cdot}31 - 0{\cdot}75\,(\log d + \delta \log d - 0{\cdot}50).$$

In Figure 1 the means given in the tables I *A*, II *A*, III *A*, IV, V*a* and VI *a*, after the application of the corrections given in the last two columns of Table VII, are plotted, those of the left hand halves of the tables (arrangement according to magnitude) being represented by dots and those of the right hand halves (arrangement according to diameters) by circles. The means used for the derivation of the formulas (4), (5*a*), (6), (7), (8 *b*) and (9 *b*) are indicated by crosses. It will be seen that, although the dots and the circles deviate systematically from the mean curve (the dots being too high in the right hand bottom corner and too low at the other end, and the circles deviating in the opposite way), the crosses lie very accurately on the straight line. A line drawn under an angle of 45° ($1 - \beta = 1$) would not represent the observations satisfactorily. In the Figure 7 of HUBBLE's paper (p. 25) all individual nebulae are plotted, including the elliptical and irregular ones, but the spirals form a very large majority. It can easily be verified that, if in this figure we draw a line at an angle of 37° (0·75 = tan 36°·9) passing just below the Andromeda nebula ($m = 5{\cdot}0$, $\log d = 2{\cdot}25$), the whole of the observations, with the only exception of the two Magellanic Clouds, are better represented than by the line at 45° drawn by HUBBLE.

5. For the *elliptical nebulae* HUBBLE remarks that more consistent results are found when the *minor* diameter $b = d\,(1 - e)$ is used instead of the major diameter d. If we suppose that 0·7 is the largest ellipticity occurring amongst these nebulae, then those classed as $E7$ all have the true ellipticity 0·7. Of those classed as $E6$ some have the true ellipticity 0·6, and some 0·7. From the Table XI given on page 30 of HUBBLE's paper we find that 3·6 out of 6·0, or 60 °/₀ have the ellipticity 0·7 and 2·4 or 40 °/₀ the

TABLE VIII. Observed and true ellipticity.

Class	$\log(1-e)$	$\log(1-e_0)$	$\log(1-e')$
E 0	·00	—·07	—·04
1	—·05	—·19	—·10
2	—·10	—·22	—·16
3	—·16	—·28	—·22
4	—·22	—·33	—·28
5	—·30	—·40	—·35
6	—·40	—·47	—·43
7	—·52	—·52	—·52

ellipticity 0·6. The average ellipticity is therefore 0·66. Similarly of those classed $E5$, 36 °/₀ have the ellipticity 0·7, 25 °/₀ 0·6 and 39 °/₀ 0·5, the average being 0·60. In this way average true ellipticities e_0 of the different classes were determined. We find the values given in the third column of Table VIII, the observed ellipticity e being given in the second column.

We may suppose that the average surface brightness of all elliptical nebulae is the same. In that case the total light is proportional to the product of the apparent major and minor axes, and consequently

$$H_1 = \frac{m}{5} + \log d + \frac{1}{2}\log(1 - e)$$

must be constant. If, on the other hand, the nebula is transparent, its total light is proportional to the volume, and

$$H_2 = \frac{m}{5} + \log d + \frac{1}{3}\log(1 - e_0)$$

should be a constant. In Figure 2 the values of H_1 are represented by circles and those of H_2 by squares It will be seen that neither of these is constant. We find, however, a pratically constant value for

$$H_3 = \frac{m}{5} + \log b,$$

both when we take $b = d\,(1 - e)$ or $b = d\,(1 - e_0)$. This seems to prove that the luminosity for unit of volume (or the surface luminosity, if we do not wish to admit a transparency of the nebulae), is systematically correlated with the ellipticity. The points in Fig. 2 represent the values of H_3 if we take $b = d\,(1 - e')$, e' being an intermediate value between e and e_0, as given in the last column of Table VIII.

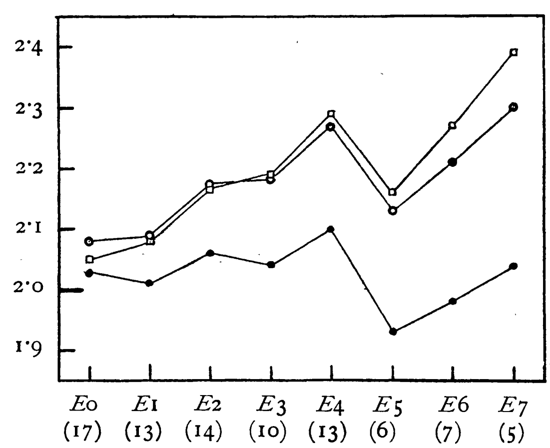

FIGURE 2. Elliptical nebulae. H_1, H_2 and H_3 (The figures in parentheses are the numbers of nebulae in each class).

It was found that the residuals are somewhat smaller if we use e' than for either e or e_0. The difference between e, e' and e_0 is, however, not large, and it is

of no great importance which of the three is used. I have adopted the values e', and by means of these reduced all diameters d given by HUBBLE to b, after which reduction all elliptical nebulae were treated as being of one class. We have then:

TABLE IX *A*. Elliptical nebulae.

Nr	$m/5$	log b	Nr	log b	$m/5$
5	2·648	−·574	7	−·740	2·558
11	2·540	−·440	7	−·554	2·452
14	2·448	−·464	15	−·424	2·444
17	2·393	−·326	12	−·332	2·385
11	2·258	−·203	18	−·215	2·266
12	2·138	−·149	9	−·092	2·240
10	1·978	+·053	11	+·047	2·052
5	1·840	+·188	6	+·228	1·886

Combining the first two and the last two means with weights proportional to the number of objects, we have the general means:

TABLE IX *B*. Elliptical nebulae.

$m/5$	log d
2·539	−·564
2·447	−·444
2·389	−·329
2·262	−·209
2·189	−·120
1·962	+·104

From these we find

$$(11)\ (\textit{Ell.})\qquad \frac{m}{5} = 2{\cdot}298 - (0{\cdot}87 \pm {\cdot}04)\,(\log b + {\cdot}260).$$

Although the probable error is probably not a true measure of the reliability, the difference of the coefficient, as well from the theoretical value 1 as from the value 0·75 for the spirals, is so large that it must be taken as real.

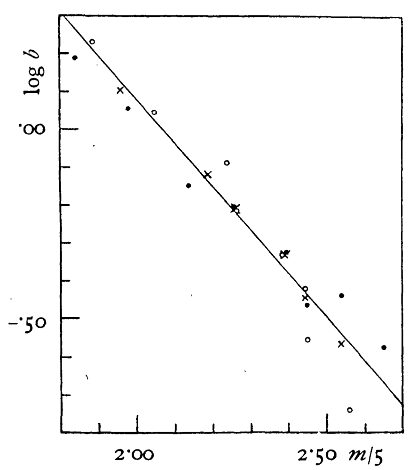

FIGURE 3. Elliptical nebulae. Correlation between $m/5$ and log b.

The means of Table IX *A* are plotted in Figure 3, those in order of magnitude (left hand half of the table) as dots, those in order of diameter as circles. The general means of Table IX *B*, from wich the formula (11) was derived, are represented by crosses.

6. *Irregular nebulae.* The number of these available for discussion is very small. To the 11 given in HUBBLE's Table IV I have added N. G. C. 6822, investigated by HUBBLE in *Mt. Wilson Contrib.* 304, and the two Magellanic Clouds. We then have the following in objects, which have in Table X been arranged in order of decreasing diameter.

TABLE X. Irregular nebulae.

Name	log d	$m/5$	p	Resid.
L. M. C.	2·64	0·10	3	−·01
S. M. C.	2·34	0·30	3	−·09
N. G. C. 6822	1·30	1·60 [1])	2	+·24
4656	1·30	2·30	0	[+·94]
4214	·90	2·26	0	[+·52]
3034	·85	1·80	1	+·02
4449	·65	1·90	1	−·07
3077	·48	2·28	1	+·15
4753	·43	2·28	1	+·11
4618	·40	2·46	1	+·26
5363	·20	2·22	1	−·17
3729	·17	2·36	1	−·06
2968	·08	1·52	0	[−·94]
5144	−·30	2·56	1	−·29

[1]) The photographic magnitude is 9·0. A colour index of 1·0 has been adopted.

No means were formed, but a straight line was determined directly by HERTZSPRUNG's method, both including all objects, and rejecting the three most discordant ones. The two straight lines were practically identical. The last solution, of which the residuals $m/5 - (m/5)_0 + 0{\cdot}93\,(\log d - \log d_0)$ are given in the table, was

$$(12)\ (\textit{Irr.})\qquad m/5 = 1{\cdot}39 - (0{\cdot}93 \pm {\cdot}15)\,(\log d - 1{\cdot}27).$$

Owing to the small number of objects on which the determination rests, the difference of the coefficient from unity is much smaller than its uncertainty. It has, however, been used as it stands.

Figure 4 contains the individual irregular nebulae as given in Table X.

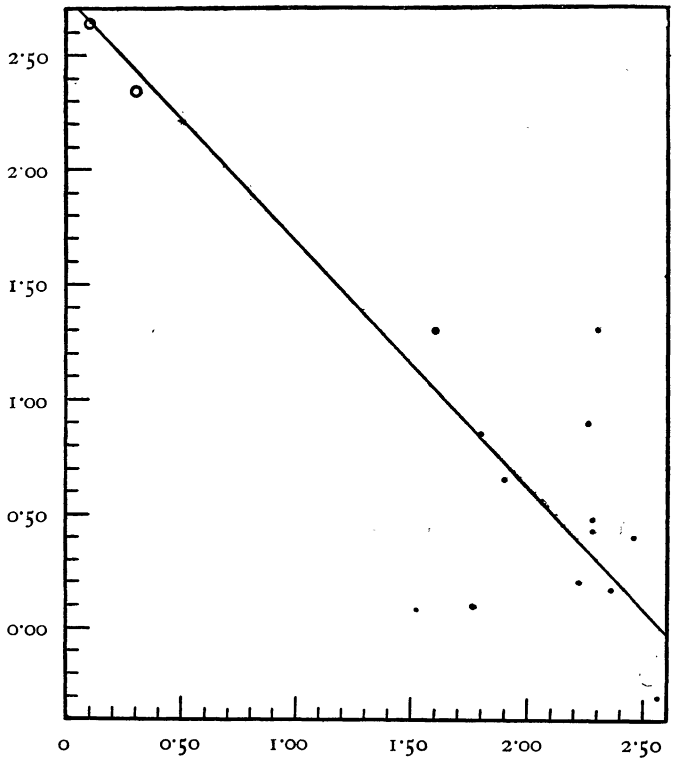

FIGURE 4.

Irregular nebulae. Correlation between $m/5$ and log d.

7. For all nebulae the coefficient β has been found positive, i.e. the increase of magnitude (decrease of brightness) with decreasing diameter is stronger than its theoretical value. It should be noted that the effect of an absorption of light in space would be in the opposite direction. For the spirals we have $\beta = +0{\cdot}25$, for the elliptical nebulae $\beta = +0{\cdot}13$. Both these values, especially that for the spirals, are considerably larger than the uncertainty of their determination. The only possible explanation seems to be by systematic errors in the scale of estimated magnitudes and diameters, the small and faint nebulae being estimated too bright, or too small, or both. It appears, indeed, a priori rather improbable that the diameters of the large and the small nebulae should conform to a consistent scale, as a direct comparison between so widely dissimilar objects must be very difficult. A method to derive reliable comparisons of the diameters of small and large objects may be by plates taken with long and short focal distances, so that the *apparent* size of the objects to be compared is the same. The angular diameters would then be in inverse ratio of the focal lengths of the telescopes. Similar, and perhaps even larger, systematic errors must exist in the scale of magnitudes. These could perhaps be diminished by comparing extrafocal images of the small objects with focal images of the larger ones. Here also, perhaps, the comparison of exposures taken with different focal distances may be used, if the ratio of the light power of the telescopes can be determined sufficiently accurately. Also exposures of different duration may be helpful.

The problem of the scale of the diameters and the magnitudes of the nebulae is a very important one, to which it is hoped practical astronomers will give all the attention that it deserves.

8. The most reliable existing determinations of distances of individual extragalactic nebulae have been based on the apparent magnitudes of objects appearing in them; cepheids, novae, the brightest stars. Other determinations are based on the total magnitude of the nebula, on observed rotation, on the apparent separation of nuclei in the spiral arms, and on other more or less hypothetical methods. In the present discussion I have only used those distances which have been determined from cepheids, novae or the brightest stars. These are collected in Table XI. A convenient unit in which to express these distances is 10^{24} cm. $= 1{\cdot}06 .\ 10^6$ lightyears $= 3{\cdot}24 .\ 10^5$ parsecs $= 6{\cdot}69 .\ 10^{10}$ astronomical units, which may be denoted by "A" [1]), as it is the distance of the nebula in Andromeda. The distances given in the column (*a*) are from different sources. Those of N.G.C. 224 and 4486 were determined by LUNDMARK, *M.N.* lxxxv, pp. 866—894 (1925), those of the two Magellanic Clouds are SHAPLEY's distances, while that given for 7619 is the distance estimated by HUBBLE for the cluster to which this nebula belongs (*Proc. Nat. Acad.* **15**, p. 173). The column (*b*) contains the distances derived by LUNDMARK, *Arkiv for Mat., Astr. o Fys.* 20 *B*, Nr. 3 (1927) and column (*c*) gives HUBBLE's distances (*Proc. Nat. Ac. Wash.* **15**, 3. p. 169, 1929). The next column contains the adopted mean with its adopted weight. For the sake of comparison I have added in the column headed "LUNDMARK" the distances derived from the curves given by LUNDMARK in *Ark. f. Mat., Astr. o Fys.* 21 *A*, Nr. 9 (1928). These curves give the distance as a function of the magnitude for the different types. The type is also given in Table XI, taken from *Veröffentlichungen der Sternwarte zu Heidelberg*, Band IX. Figures in parentheses were derived by extrapolation from the curves. For 4449, of wich no type is given in the Heidelberg publication, and 6822, which does not occur there, the curve „Mag"

[1]) No confusion with the Angström unit, Å is to be feared. The ratio of the two units is 10^{32}.

of LUNDMARK was used. It will be seen that most of these distances are considerably larger than those in the columns (*a*), (*b*) and (*c*). The only exception is formed by the four elliptical nebulae 4382, 4472, 4486, 4649, belonging to the Virgo cluster. SHAPLEY's estimate for the distance of this cluster is 9·5 *A*.

TABLE XI. Distances of extragalactic nebulae.

N. G. C.	Distances (*a*)	(*b*)	(*c*)	Adopted *r*	*p*	LUNDMARK type	*r*	log *d*	*m*/5	log *r*
Sb	*A*	*A*	*A*	*A*			*A*			
224	1·4	1·0	0·9	1·0	4	*s*		2·25	1·00	·00
1068		2·5	3·1	2·8	2	*t*	5·5	·40	1·82	·45
3031		4·0	2·7	3·3	2	*w*—*r*	(2·4) — (3·7)	1·20	1·66	·52
3627		4·0	2·7	3·3	2	*r*	5·2	·90	1·82	·52
4151			5·2	5	1	*s*—*l*	22·6 —	·40	2·40	·72
4258			4·3	4	1	*r*	(4·5)	1·30	1·74	·63
4450		6·3		6	1	*s*	11·9	·57	2·12	·80
4736		2·3	1·5	1·9	2	*t*—*n*	(4·0) —	·70	1·68	·28
4826			2·7	3	1	*n*		·90	1·80	·43
5055		6·3	3·4	4·8	2	*s*	7·5	·90	1·92	·68
7331		5·0	3·4	4·2	2	*s*	10·9	·95	2·08	·62
Sc										
253		3·6		3½	1			1·34	1·86	·56
598		1·0	0·8	0·9	3	*r*—*w*		1·78	1·40	— ·055
2403		2·3		2	1	*r*	(4·5)	1·20	1·74	·30
2903		5·0		5	1	*w*—*t*	3·4—5·5	1·04	1·82	·70
4254		4·0		4	1	*w*	6·2	·65	2·08	·60
4321		4·6		4½	1	*w*	6·5	·70	2·10	·66
4490		4·6		4½	1	*v*—*w*	— 5·7	·60	2·04	·66
5194		2·3	1·5	1·9	2	*w*		1·08	1·48	·28
5236		4·2	2·7	3·4	2			1·00	2·08	·53
5457		2·0	1·4	1·7	2	*w*	4·9	1·34	1·98	·23
Ell.								log *b*		
205		1·0		1	1	*s*	6·5	·70	1·86	·00
221		1·0	0·9	1·0	4	*g*	(1·6)	·26	1·76	·00
4382			6·2	6	1	*g*	2·8	·20	2·00	·78
4472			6·2	6	1	*g*	(1·6)	·20	1·76	·78
4486	8·0	6·3	6·2	6·8	2	*f*	0·9	·26	1·94	·83
4649			6·2	6	1	*g*	2·3	·14	1·90	·78
7619	(21·5)					*f*	2·4	— ·37	2·36	(1·33)
Irr.								log *d*		
L. M. C.	·106	·06	·105	0·106	4			2·64	0·10	— ·975
S. M. C.	·097	·20	·099	0·098	4			2·34	0·30	— 1·01
4214			2·5	2½	1	*r*—*l*	14·2 —	·90	2·26	·40
4449		2·9	1·9	2·4	2	Mag.	3·0	·65	1·90	·38
4485		4·6		4½	1	*r*?	(25)	·18	2·52	·65
6822		1·15	0·66	0·9	3	Mag.	(1·4)	1·30	1·60	— ·05

9. Table XI also contains the values of log *d* and *m*/5 (without the corrections of Table VII), and in the last column the logarithms of the adopted values of the distance log *r*. For the elliptical nebulae $\log b = \log d + \log (1 - e')$ (table VIII) has been given. From these we have derived linear formulas:

$$\log r_d = c - (1 - \gamma) \log d$$

$$\log r_m = (1 - \varkappa)\frac{m}{5} - b. \tag{13}$$

HERTZSPRUNG's method was used, with the weights of log *r* given in Table IX. The following results were found.

Sb: $\log r_d = 0·44 - (0·34 \pm ·07)(\log d - 1·11)$
$\log r_m = 0·44 + (0·66 \pm ·07)\;(m/5 - 1·70)$

Sc: $\log r_d = 0·36 - (0·66 \pm ·10)(\log d - 1·18)$
$\log r_m = 0·36 + (0·95 \pm ·22)\;(m/5 - 1·80)$

Irr: $\log r_d = -0·42 - (0·75 \pm ·06)(\log d - 1·75)$
$\log r_m = -0·42 + (0·72 \pm ·05)\;(m/5 - 1·00)$

For the elliptical nebulae the data are not sufficient to derive a formula.

The probable errors of the coefficients cannot be assumed to be a true measure of the uncertainty, which must be much larger. It appears very improbable that the large deviations of nearly all coefficients from the theoretical value 1 can be real. It must partly be due to systematic errors in the scales of the diameters, magnitudes and distances, and partly to an effect of selection.

We have also combined the spirals *Sb* and *Sc*, applying the corrections $-\cdot05$ to $\log d$ and $m/5$ of *Sc* to reduce them to *Sb* (see Table VII). Calling the corrected values $\log d'$ and $m'/5$, we find

$$Sb + Sc: \quad \log r_d = 0{\cdot}40 - (0{\cdot}42 \pm \cdot02)(\log d' - 1{\cdot}12)$$
$$\log r_m = 0{\cdot}40 + (0{\cdot}68 \pm \cdot03^5)(m'/5 - 1{\cdot}72)$$

Comparing the formulas (13) with

$$(3) \qquad \frac{m}{5} = a - (1-\beta)\log d$$

it appears that we must have

$$(14) \qquad \begin{aligned} c + b &= (1-\varkappa)\, a \\ \gamma &= \beta + (1-\beta)\,\varkappa \end{aligned}$$

The values of b, c, γ and $\varkappa$ directly derived do not satisfy these conditions. The first of (14) can, by using the second, be written

$$(14') \qquad (1-\beta)\log d_0 + (m/5)_0 = a,$$

$\log d_0$ and $(m/5)_0$ being the weighted means of the values of $\log d$ and $m/5$ respectively that were used in deriving the formulas (13), the corresponding mean for $\log r$ being $\log r_0$. We have:

TABLE XII. Coefficients of equations.

Class	a	β	$\log r_0$	$\log d_0$	$(m/5)_0$	γ	$\varkappa$	Adopted $\varkappa$	Adopted γ	Adopted $\log d_0$	Adopted $(m/5)_0$
Spirals	2·68	0·25	0·40	1·12	1·72	0·58	0·32	0	0·25	1·17	1·80
Elliptical	2·07	0·13	0·80	—	—	—	—	0	0·13	—·03	2·10
Irregular	2·57	0·07	—0·42	1·75	1·00	0·25	0·28	0·09	0·15	1·72	0·97

For the elliptical nebulae no formula was derived. We have, however, for $\log r_0$ taken the mean of the four nebulae belonging to the Virgo cluster. The adjustment of the values of γ, $\varkappa$, $\log d_0$ and $(m/5)_0$ to satisfy (14) and (14′) is largely arbitrary. Various adjustments were tried, starting from different assumed values of $\varkappa$. The representation of the observed data was not much different for these different formulas, though, of course, not quite so good as with the directly determined values of γ and $\varkappa$. These latter, however, have very little weight, and cannot be taken as real, as has already been pointed out. They have practically been discarded altogether, and I have finally decided to give preference to a small, or zero, value of $\varkappa$. The adopted values ares given in the second half of Table XII. It will be seen that the corrections to $\log d_0$ and $(m/5)_0$ are insignificant. The final formulas are thus

$$(15) \quad \textit{Spirals:} \quad \log r_d = 1{\cdot}28 - 0{\cdot}75 \log d'$$
$$\log r_m = -1{\cdot}40 + m'/5,$$

where $\log d'$ and m' include the corrections of Table VII,

$$(16) \quad \textit{Elliptical:} \quad \log r_d = 0{\cdot}77 - 0{\cdot}87 \log b$$
$$\log r_m = -1{\cdot}30 + m/5,$$

where $\log b = \log d + \log(1-e')$, the values of $\log(1-e')$ being taken from Table VIII,

$$(17) \quad \textit{Irregular:} \quad \log r_d = 1{\cdot}04 - 0{\cdot}85 \log d$$
$$\log r_m = -1{\cdot}30 + 0{\cdot}91\, m/5.$$

In the figures 5 to 9 the broken lines are those derived directly from the data, the full lines are the final formulas (15), (16), (17).

10. It is not easy to from a judgment concerning the reliability of the distances derived by these formulas. We must distinguish between the accidental uncertainty, due to accidental errors in the determination of the individual magnitudes and diameters and the spreading of the true magnitudes and diameters round their average value, and the systematic uncertainty, due on the one hand to the *scale* of the observed magnitudes and diameters, and on the other hand to that of the distances on which the formulas were based.

The accidental uncertainty can be derived by comparing the values of $\log r_d$ and $\log r_m$ for a number of nebulae. I find in this way that it is of the order of $\pm\cdot05$ or, at the utmost, $\pm\cdot06$ (p.e.) for the mean of $\log r_d$ and $\log r_m$ of one nebula.

The systematic uncertainty is more important. The fact that for all the coefficients γ and $\varkappa$ large positive values were found, seems to show that the distances of the small and faint nebulae used in the investigation have been systematically too small. These are mostly distances based not on cepheids or novae, but on "brightest stars" which, as SHAPLEY has pointed out, may perhaps not be stars but star clusters. The effect of this has, however, largely been eliminated from the final formulas (15), (16), (17), by the choice of $\varkappa = 0$

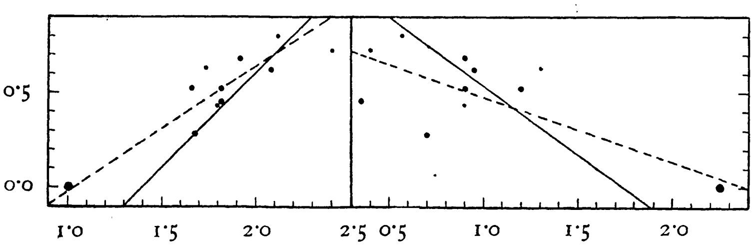

FIG. 5. Spirals *Sb*.

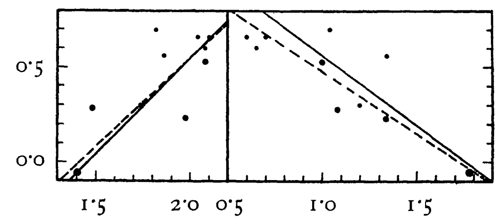

FIG. 6. Spirals *Sc*.

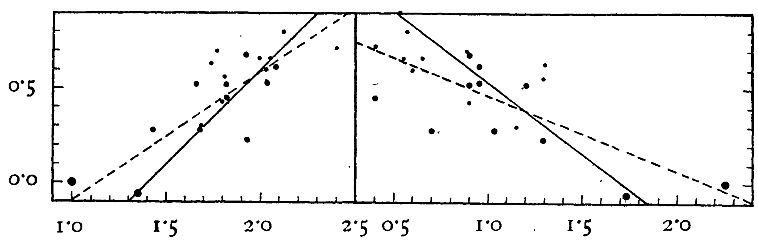

FIG. 7. All Spirals (*Sb* + *Sc*).

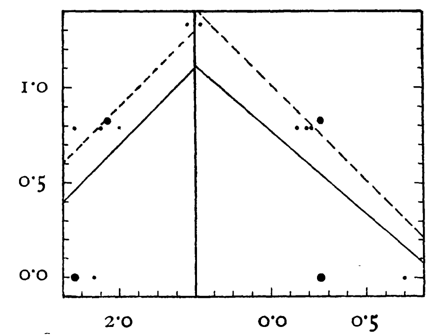

FIG. 8. Elliptical.

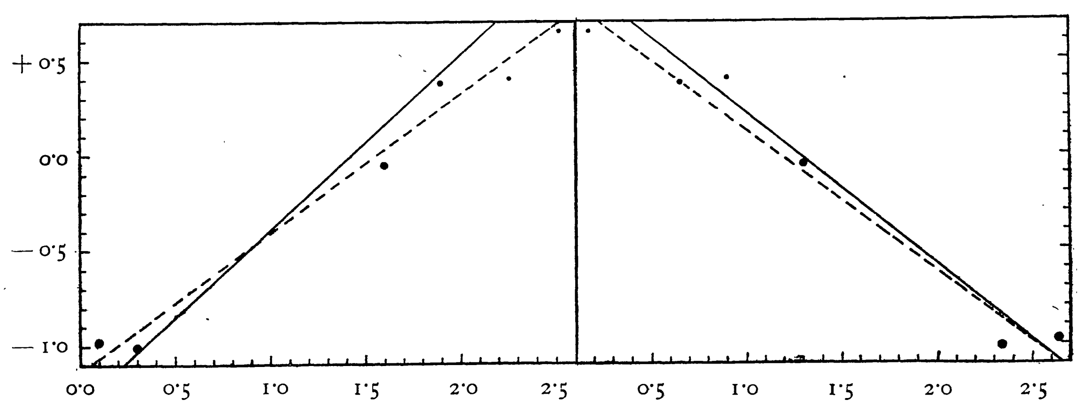

FIG. 9. Irregular Nebulae.

FIGURES 5 to 9. Relation between distance and magnitude and between distance and diameter.

The vertical coordinate in all figures is $\log r$. The horizontal coordinate in the left hand halves of the figures is $m/5$, in the right hand halves it is $\log d$ for the spirals and irregular nebulae, and $\log b$ for the elliptical ones. The diameters of the dots are proportional to the square roots of the weight of $\log r$.

The broken lines are those derived from each group separately. For the elliptical nebulae (Fig. 8) they have been drawn at an angle of 45° through the centre of gravity of the four nebulae near the centre of the diagram. The full lines are the finally adopted formulae (15), (16), (17).

(or small) and the adjustment of γ to the value of β derived previously from all HUBBLE's nebulae. As interpolation formulas within the range of distances on which they are based (say from about 2 A tot 5 or 6 A) our formulas are probably trustworthy, but for *extrapolation* beyond that range their reliability must decrease rapidly.

11. The observed radial velocities of extragalactic nebulae have been collected in Table XIII. In the first column the class (ellipticity) of the elliptical nebulae has been given in parentheses. The next column gives the observed velocity V; ΔV is the correction for the motion of the sun, for which I am indebted to Dr. OORT. The adopted velocity is STRÖMBERG's, viz: 286 km./sec. towards $\alpha = 314°$, $\delta = + 66°$, coinciding nearly with that due to the rotation of the galaxy.

Then $\varphi = \log [1000\,(V + \Delta V)/c]$, c being the velocity of light.

The next columns give $\log d'$ and $m'/5$, $\log d'$ and

m' being HUBBLE's $\log d$ and m with the corrections of Table VII. For the elliptical nebulae $\log b = \log d + \log(1-e')$ is given, $\log(1-e')$ being taken from Table VIII. Then follow $\log r_d$ and $\log r_m$ computed by the formulas (15), (16), (17) and, when available $\log r_0$, i.e. the adopted $\log r$ from Table XI, and its weight. The finally adopted $\log r$, and the corresponding numerical value of r, are contained in the next two columns. In combining $\log r_d$, $\log r_m$ and $\log r_0$ weight 1 was assigned to $\log r_d$ and $\log r_m$.

The values of r and of $V + \Delta V$ have been plotted in figure 10, where the spirals are represented by dots, the elliptical nebulae by circles and the irregular nebulae by crosses. N.G.C. 4824 has been entered as

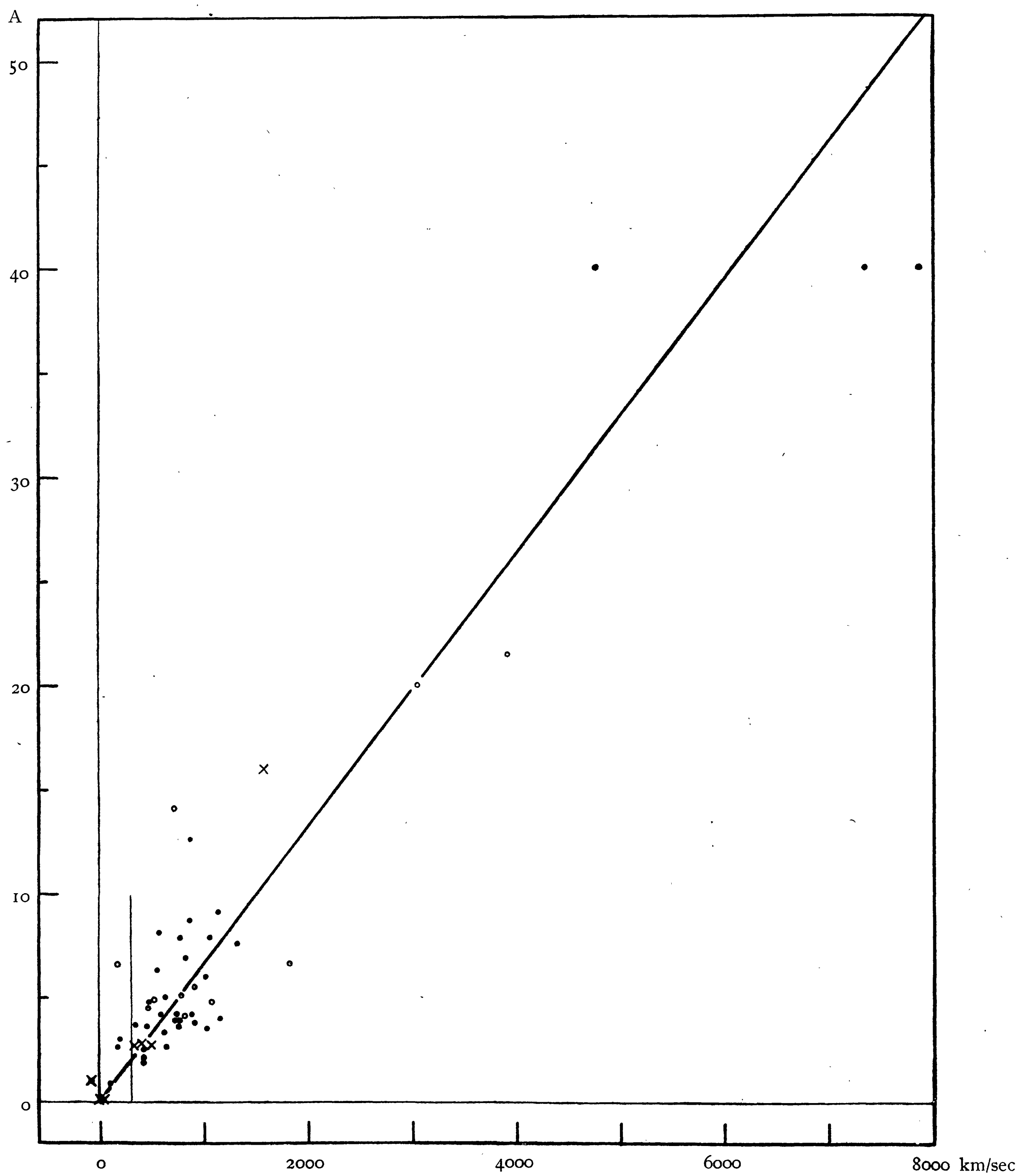

FIGURE 10. Distances and radial velocities.

TABLE XIII. Correlation between radial velocity and distance.

N. G. C.	V	ΔV	φ	log d'	$\frac{m'}{5}$	log r_d	log r_m	log r_o	p	Adopted log r	r	log $r - \varphi$	r_v
Sa	km/sec.										A		A
4526	+ 580	− 30	+ ·26	·75	2·27	+ ·72	+ ·87			+ ·80	6·3	+ ·54	3·6
2681	+ 700	+ 130	·44	·53	2·19	·88	·79			·84	6·9	·40	5·5
5866	+ 650	+ 220	·46	·53	2·39	·88	·99			·94	8·7	·48	5·7
3368	+ 940	− 50	·47	·90	2·05	·60	·65			·62	4·2	·15	5·9
4594 [2]	+ 1140	− 110	·53	·90	1·87	·60	·47			·54	3·5		6·8
Sb													
224	− 310	+ 220	—	2·25	1·00	− ·41	− ·40	+ ·00	4	·00	1·00		—
3031	− 30	+ 200	− ·25	1·20	1·66	+ ·38	+ ·26	·52	2	+ ·42	2.6		(1·1)
4826	+ 150	+ 40	− ·20	·90	1·80	·60	·40	·43	1	·48	3·0		(1·2)
4736	+ 290	+ 130	+ ·14	·70	1·68	·76	·28	·28	2	·40	2·5	+ ·26	2·8
3489	+ 600:	− 30	·28	·40	2·24	·98	·84			·91	8·1	[·63]	3·8:
5055	+ 450	+ 140	·29	·90	1·92	·60	·52	·69	2	·62	4·2	·33	3·9
3627	+ 650	− 30	·31	·90	1·82	·60	·42	·52	2	·52	3·3	·21	4·1
4258	+ 500	+ 140	·33	1·30	1·74	·30	·34	·63	1	·42	2·6	·09	4·3
2841	+ 600	+ 130	·38	·78	1·88	·70	·48			·59	3·9	·21	4·8
7331	+ 500	+ 240	·39	·95	2·08	·57	·68	·62	2	·62	4·2	·23	4·9
3623	+ 800	− 30	·41	·90	1·98	·60	·58			·59	3·9	·18	5·1
4051 [1]	+ 650	+ 130	·41	·60	2·38	·83	·98			·90	7·9		5·1
1068	+ 920	0	·48	·40	1·82	·98	·42	·45	2	·58	3·8	·10	6·0
4151	+ 950	+ 110	·54	·40	2·40	·98	1·00	·72	1	·90	7·9	·36	6·9
3227 [1]	+ 1150	− 10	·58	·48	2·40	·92	1·00			·96	9·1		7·6
4565 [3]	+ 1100	+ 60	·59	1·17	2·20	·40	·80			·60	4·0		7·8
Sc													
598 [4]	− 70	+ 170	− ·48	1·73	1·35	− ·77	− ·05	− ·05[5]	3	− ·06	0·87		(0·7)
5236	+ 500	− 160	+ ·05	·95	2·03	+ ·57	+ ·63	+ ·54	2	+ ·57	3·7	+ ·52	2·2
5194 [5]	+ 260	+ 160	·14	1·03	1·43	·52	·03	·28	2	·28	1·9	·14	2·8
5457	+ 220	+ 200	·14	1·29	1·93	·31	·53	·23	2	·32	2·1	·18	2·8
2683	+ 400	+ 50	·17	·95	1·93	·57	·53			·55	3·6	·38	3·0
3521	+ 730	− 100	·32	·60	1·97	·83	·57			·70	5·0	·38	4·2
278	+ 650:	+ 230	·47	·03	2·35	1·26	·95			1·10	12·6	[.63]	5·9:
5005	+ 900	+ 120	·53	·65	2·17	·79	·77			·78	6·0	·25	6·8
SBa													
1023	+ 300	+ 170	+ ·20	·56	2·13	+ ·64	+ ·73			+ ·68	4·8	+ ·48	3·2
936	+ 1300	+ 10	·64	·86	2·31	·86	·91			·88	7·6	·24	8·7
Ell.				log b									
(2) 221	− 300	+ 220	—	+ ·26	1·76	+ ·54	+ ·46	·00	4	·00	1·00		—
(0) 404	− 25	+ 195	− ·25	·07	2·22	·71	·92			+ ·82	6·6		(1·1)
(7) 3115	+ 600	− 140	+ ·18	·06	1·90	·70	·60			·65	4·5	+ ·47	3·0
(4) 4382	+ 500	+ 20	·24	+ ·20	2·00	·60	·70	·78	1	·69	4·9	·45	3·5
(4) 1700	+ 700	− 80	·38	− ·38	2·50	1·10	1·20			1·15	14·1	·77	4·8
(0) 3379	+ 810	− 50	·40	+ ·26	1·88	·54	·58			·56	3·6	·16	5·0
(0) 4486	+ 800:	− 10	·42	·26	1·94	·54	·64	·83	2	·71	5·1	[·29]	5·2:
(1) 4472	+ 850	− 30	·43	·20	1·76	·60	·46	·78	1	·61	4·1	·18	5·4
(7) 4111	+ 800	+ 120	·49	·02	2·02	·75	·72			·74	5·5	·25	6·2
(2) 4649	+ 1090	− 10	·55	·14	1·90	·65	·60	·78	1	·68	4·8	·13	7·1
(4) 584	+ 1800	+ 10	·78	+ ·02	2·18	·75	·88			·82	6·6	·04	12·0
(0) 6359 [1]	+ 2800	+ 260	1·01	− ·74	2·40	1·41	1·10			1·30	20		20·4
(3) 7619 [6]	+ 3780	+ 130	1·11	− ·37	2·36	1·09	1·06	(1·33)		1·33	21·5	[·22]	25·7

TABLE XIII (*continued*).

N. G. C.	V	ΔV	φ	log d	$\frac{m}{5}$	log r_d	log r_m	log $r_\circ$	p	Adopted log r	r	log $r-\varphi$	r_v
Irr.													
S. M. C.	+ 163	− 172	—	2·34	0·30	− ·95	− 1·03	+ 1·01	4	− 1·01	0·098		—
L. M. C.	+ 276	− 235	− ·86	2·64	0·10	− 1·20	− 1·21	− ·98	4	− ·98	0·106		(0·3)
6822	− 130	+ 40	—	1·30	1·60	− ·06	+ ·16	− ·05	3	− ·01	0·98		—
4449	+ 200	+ 130	+ ·04	·65	1·90	+ ·49	·43	+ ·40	1	+ ·44	2·7	+ ·40	2·2
4214	+ 300	+ 100	·12	·90	2·26	·28	·76	·38	2	·45	2·8	·33	2·6
3034	+ 290	+ 210	·22	·85	1·80	·32	·34	·65	1	·44	2·7	·22	3·3
4824 [7])	+ 1500	+ 80	+ ·72	− ·20	2·5:	see note				+ 1·2	16		10·5
4865 [8])	+ 4700	+ 70	+ 1·20	− ·30	3·0:	+ 1·50	+ 1·6			+ 1·6	40		31·6
4853 [8])	+ 7300	+ 60	1·39	− ·52	3·0:	1·67	1·6			1·6	40		49·0
4860 [8])	+ 7800	+ 70	1·42	− ·30	3·0:	1·50	1·6			1·6	40		52·5

Notes to Table XIII.

[1]) N. G. C. 3227, 4051, 6359: radial velocity published after the completion of the discussions for the present paper (*Annual Report Mt. Wilson Observatory*, 1928—29, page 126).

[2]) N. G. C. 4594: type appears doubtful, WOLF has *p*, which is more like irregular than spiral.

[3]) N. G. C. 4565: "absorption very conspicuous". WOLF's type is *o*.

[4]) The companion to N. G. C. 598, N. G. C. 604, has a radial velocity of — 270 km/sec.

[5]) The companion to N. G. C. 5194, N. G. C. 5195, has a radial velocity of + 230 km/sec.

[6]) N. G. C. 7619: HUBBLE's distance of the cluster, of which this nebula is one of the largest and brightest members, has been adopted.

[7]) N. G. C. 4824: type unknown: if *Sb*, log $r_d = 1·4$, log $r_m = 1·1$; if *E*2, log $r_d = 1·1$, log $r_m = 1·2$; if irregular, log $r_d = 1·2$, log $r_m = 1·1$.

[8]) N. G. C. 4853, 4860, 4865: HUBBLE's estimate for the distance of the Coma cluster, in which these nebulae are situated, is 50 *A*. This would bring 4853 and 4860 exactly on the curve. The diameters, and especially the magnitudes, are very uncertain. The type has been assumed to be *Sb*. For 4865 it may be *E*6, which would make log $r_d = 1·38$, log $r_m = 1·7$. It appears probable that 4865 does not belong to the Coma cluster, its distance being about 30 *A*, and that HUBBLE's distance of 50 *A* for the cluster is very nearly right. The probable error of the radial velocity of each of these three nebulae is ± 250 km/sec.

an irregular nebula. N.G.C. 221, the companion to the Andromeda nebula N.G.C. 224, has been omitted from the diagram.

It has been remarked by several astronomers that there appears to be a linear correlation between the radial velocities and the distances. If this is so, the difference log $r - \varphi$ must be constant. This difference is given in the last column but one of Table XIII. Omitted are:

1. the three nebulae N.G.C. 3227, 4051, 6359, of which the radial velocity was only published after the present discussion had been completed. It will be seen that they confirm the general result.

2. all radial velocities smaller than 300 km./sec., as the accidental peculiar velocity is too large a part of the whole. The choice of the limit 300 km./sec., or $\varphi = 0$, is, of course, rather arbitrary, but it has no appreciable influence on the result.

3. the nebulae N.G.C. 4565 and 4594, both of which have peculiarities, which make the determination of the distance from the diameter and the magnitude peculiarly uncertain.

4. the nebulae N.G.C. 278, 3489, 4486 of which the radial velocity is exceptionally uncertain.

5. N.G.C. 7619, of which the distance is too uncertain.

For the others we have:

TABLE XIV. Values of log $r - \varphi$

Class	Nr.	log $r - \varphi$
Sa	4	+ ·39
Sb	9	+ ·22
Sc	6	+ ·31
SBa	2	+ ·36
Ell.	8	+ ·30
Irr.	3	+ ·32
All spirals	21	+ ·294
All nebulae	32	+ ·293 ± ·020 (p.e.)

Of the 32 nebulae used, 22 have distances between 2 and 5 *A*, and only 2 distances exceed 8 *A*. It is thus not possible to derive a straight line by HERTZSPRUNG's method. The small probable error of the final mean (which was derived from the individual residuals),

shows, however, the reality of the correlation. The straight line is thus practically determined by the origin of coordinates and the centre of gravity of the nebulae used.

We have thus

$$\log r = \varphi + 0{\cdot}30$$

or

$$(18) \qquad r = 2000 \frac{V}{c} = 0{\cdot}0067\ V,$$

V being the radial velocity corrected for the solar motion (rotation of the galaxy). The values of r computed by (18) have been called r_v. They are given in the last column of Table XIII. It cannot reasonably be doubted that for large distances these are more reliable than either r_d or r_m. As a formula of interpolation within the range of distances from which it was derived, i.e. from about 2 *A* to 5 *A*, or perhaps to 7 or 8 *A*, (18) is systematically equally reliable as (15), (16), (17). The accidental errors of small distances derived from (18) will be larger than from (15), (16), (17) on account of the greater spreading of the individual velocities, as compared with the diameters and the magnitudes. The probable error of one value of $\log r - \varphi$ is about $\pm 0{\cdot}11$, of which not more than $\pm{\cdot}05$ is due to $\log r$. It follows that the spreading of the velocities corresponds to a p.e. of $\pm{\cdot}10$ in φ, which, at the average distance of these nebulae, is equivalent to $\pm{\cdot}09$ or $\pm{\cdot}10$ in r_v, or to about ± 140 km./sec. in the radial velocity. HUBBLE's estimate is ± 150 km./sec. (*Mt. Wilson Report*, 1928—29, page 126).

The value of the formula (18) as an extrapolation formula for large distances depends on the uncertainty of the coefficient, and this, of course depends on the scale of distances. We can say that the scale of distances which is used up to 5 or 6 *A*, and on which the derivation of (18) depends, is by (18) continued conformally to larger distances, with an error probably not exceeding 10 %.

12. The only acceptable explanation of these large receding velocities so far proposed is by the inherent expanding tendency of space which follows from the solution, that was called solution (*B*) in my paper of 1917 [1]), of EINSTEIN's differential equations for the inertial field. It will be remembered that these equations admit two, and only two [2]) solutions which are static and isotropic. If we denote the line element of the three-dimensional elliptical (or spherical) space of unit radius of curvature by $d\sigma^2$ thus

$$d\sigma^2 = \frac{dr^2}{1 - r^2} + r^2 (d\psi^2 + \sin^2\psi\, d\theta^2),$$
$$= d\chi^2 + \sin^2\chi\,(d\psi^2 + \sin^2\psi\, d\theta^2),$$

where $\sin\chi = r$, or another equivalent expression in other coordinates, we can write for the line element of the four dimensional time-space

$$(19) \qquad ds^2 = - R^2\, d\sigma^2 + f\, dt^2.$$

The two solutions are

$$(20) \quad \begin{array}{lll} (A) & R = \text{const.} & f = \text{const.} = c^2 \\ (B) & R = \text{const.} & f = c^2 (1 - r^2) = c^2 \cos^2\chi. \end{array}$$

In solution (*A*) the path of a material point is a straight line described with a constant velocity. Thus the *systematic* velocity (apart from the peculiar velocities) is zero. In (*B*) this path is a hyperbola, and the radial component of the velocity is given by

$$\frac{R^2 V^2}{c^2 \mathrm{r}^2} = \left(1 - \frac{\mathrm{r}_0^2}{\mathrm{r}^2}\right)\left(1 + \frac{R^2 v_0^2}{c^2 \mathrm{r}^2}\right),$$

where r_0 and v_0 are the minimum radius vector, and the transverse velocity $\mathrm{r}_0\, d\theta/dt$ at this distance [1]). Thus for large distances the ratio $R^2 V^2/c^2\mathrm{r}^2$ rapidly approaches the limit unity. The time spent by a body in the neighbourhood of its minimum distance is only a short fraction of its whole life, and during the major part of its course the value of $R^2 V^2/c^2\mathrm{r}^2$ does not differ systematically from 1. There is, however, no apparent reason in this theory why V should be positive rather than negative.

If by ρ_0 we denote the average invariant density of matter in space (the pressure being neglected, or assumed to be zero) [2]), the solution (*A*) leads to a relation between ρ_0 and R. In (*B*) we have $\rho_0 = 0$. We have thus for the two cases, $\varkappa$ being the constant of gravitation:

$$(21) \quad \begin{array}{lll} (A) & \varkappa\rho_0 = \dfrac{2}{R^2}, & \dfrac{V}{r} = 0, \\ (B) & \varkappa\rho_0 = 0, & \dfrac{V^2}{c^2 r^2} = \dfrac{1}{R^2}. \end{array}$$

The last formula in (*B*) is only approximate, neglecting the square and higher powers of r/R. It has no meaning to take the square of r/R into account, as r is only defined to the first order. In both cases the formula for V gives statistical values, from which the velocities of individual bodies may deviate more or less.

Now we have found in the preceding article a

[1]) On EINSTEIN's theory of gravitation, and its astronomical consequences. Third paper. *M. N.* lxxviii, **1**, pp. 3—28 (1917).

[2]) The solution given by LEVI CIVITA on the pages 435—436 of his book *Absolute differential Calculus* leads to a dependency of the mean invariant density ρ_0 and the pressure p on the radius-vector, which is too artificial to be physically acceptable.

[1]) See my quoted paper, pp. 16, 18, 19. The coordinate r here is $\mathrm{r} = \tan\chi$, but very similar formulae are found for $r = \sin\chi$, or other coordinates.

[2]) The pressure is not necessarily zero, neither in (*A*) nor in (*B*). In the formulas (21) we can replace $\varkappa\rho_0$ by $\varkappa(\rho_0 + 4p)$.

definite value for V/r. On the other hand we can make an estimate of the mean density ρ_0, supposed to be constant. If we take the average density to be: one nebula in a volume of one A^3, and for the average mass of one spiral we take 10^{11} ⊙, (estimate by Dr. OORT), we find, in C. G. S. units, $\rho_0 = 2 \cdot 10^{-28}$, or

(22) $$\varkappa\rho_0 = 3{\cdot}7 . 10^{-55}.$$

The value of V/r found above is $V/cr = 1/2000$, or in C. G. S. units

(23) $$\frac{V}{cr} = 0{\cdot}5 . 10^{-27}.$$

This would thus, according to (21, B) give

(24) $$R_B = 2.10^{27}\ cm = 2000\ A.$$

The solution (B) can, however, only be admitted if the value of ρ_0 is so small, that $\rho_0 = 0$ can be considered to be a good approximation. Now, in the solution (A) we would from the value (22) of $\varkappa\rho_0$ find

(25) $$R_A = 2{\cdot}3.10^{27}\ \text{cm.}.$$

The numerical value (24) of R_B can be assumed to be fairly reliable. It agrees exactly with the value found by HUBBLE in *Proc. Nat. Acad.* **15**, 3 (1929). The probable error of the coefficient 2000 in (18) can be taken to be not more than, say, 10 °/₀. The scale of distances may be in error by a larger amount than this, but taking everything into account, it appears safe to assert that the probable error of (24) is not more than 20 °/₀ or 25 °/₀. The value (25) is much more uncertain. On the one hand the distribution of extragalactic nebulae through space is far from uniform, and to take the average density in our neighbourhood as the mean density of the whole of space may not be a good approximation. On the other hand this average density in our neighbourhood is itself very uncertain. HUBBLE's value of ρ_0 as derived in *Mt. Wilson Contr.* 324 (1926) is $1{\cdot}5.10^{-31}$, which HUBBLE considers as a lower limit. His density in nebulae per unit volume is 0·3 times the value adopted here, but his adopted mass of one nebula is only $2{\cdot}6.10^8$ ⊙ instead of 10^{11} ⊙. This seems to be rather too small. The value adopted here is Dr. OORT's estimate for the mass of the galactic systeem (*B. A. N.* 132 and 133). It includes, of course, all gravitating mass: faint and dark stars, interstellar matter, dark nebulae, globular clusters, etc. It appears probable that our galactic system is somewhat larger than the average spiral nebula, and thus our estimate of the density may be too high, and consequently the value (25) of R_A too small. HUBBLE's value, $R_A = 8{\cdot}5.10^{28}$ cm, on the other hand, is most probably too large. As an extreme value within the limits of probability we may take for ρ_0 one hundredth of the value (22), and consequently for R_A ten times the value (25).

Notwithstanding the remaining uncertainty of the values (24) and (25) it is thus certain that R_A and R_B are of the same order of magnitude. Consequently, in the solution (B), $\rho = 0$ is *not* an approximation, and we thus come to the conclusion that neither the solution (A) nor (B) can correspond to the truth, (A) being excluded by the large positive systematic velocity V, which is in contradiction with the second of (21, A), and (B) by the finite density, which is excluded by the first of (21, B).

It thus appears that a static solution of EINSTEIN's field equations cannot represent the observed facts. A non-static solution is contained in a paper by Dr. G. LEMAÎTRE [1]), published in 1927, which had failed to attract my notice, but to which my attention was called by Professor EDDINGTON only a few weeks ago. LEMAÎTRE takes R and f in (19) as functions of t, instead of r. He then finds a solution

(20) (C) $$R = R(t) \qquad f = \text{const.} = c^2,$$

in which the radius of curvature increases with the time from a limiting value R_0 for $t = -\infty$ to infinity for $t = +\infty$. His theory leads to two equations, which can be written thus:

(21) (C) $$\varkappa\rho_0 = \frac{2R_0}{R^3}, \qquad \frac{V^2}{c^2r^2} = \frac{1}{3}\left(\frac{1}{R_0^2} + \frac{2R_0}{R^3}\right) - \frac{1}{R^2},$$

from which the initial value R_0 and the present value R can be derived. It will be seen that for $R = R_0$ this degenerates into (21, A), while for $R = \infty$ it is not in direct contradiction with (21, B), though not identical with it. Using the values (22) and (23) of $\varkappa\rho_0$ and V^2/c^2r^2, we find from (21, C):

(26) $$R_0 = 0{\cdot}8.10^{27}\ \text{cm.}, \qquad R = 1{\cdot}6^5.10^{45}\ \text{cm.}$$

The present radius of the universe would thus be twice the initial value.

If instead of (25) we took a ten times larger value of R_A, i. e. if we used 1/100 of the value (22) for $\varkappa\rho_0$, we would find

(26′) $$R_0 = 1{\cdot}1.10^{27}\ \text{cm}, \qquad R = 8{\cdot}4.10^{27}\ \text{cm}.$$

The total mass of the universe is $\pi^2R^3\rho_0 = 2\pi^2R_0/\varkappa$, i. e. $0{\cdot}9.10^{55}$ gr. or $1{\cdot}2.10^{55}$ gr. according to the values (26) or (26′) of R_0 respectively.

I hope to return to the discussion of this ingenious solution in a separate communication.

[1]) Un Univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques, Note de M. l'Abbé G. LEMAITRE, *Annales de la Société Scientifique de Bruxelles*, Tome XLVII, série A, première partie. Comptes rendus des séances, p. 49. Session du 25 avril 1927. Première section.

ERRATA in *B.A.N.* 184.

p. 154, upper middle part of TABLE 1, line 7, for ·47536 read ·47537

p. 155 second half op TABLE 2, line 3, for 4504· .... read 5504·